

Recent research progress on Experimental quantum simulation: Simulating many-body physics on a NMR quantum simulator

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Outline

I. Introduction

- Why quantum simulation (QS)
- Basic principle of QS
- II. Operations Interpreted for Experimental QS
	- Mapping the system
	- Initialization
	- Hamiltonian engineering
	- Measurement
- III. Towards Simulating Many-Body Physics
	- Quantum information and many-body systems
	- Quantum "baby" phase transition (Simulating Quantum magnets)
	- Simulating Exotic quantum many-body physics: Topological orders in Wen-plaquette model
- V. Conclusion

Why QS?

Simulating of quantum systems

well beyond the capacity of existing computers

3 The Puzzle: Feynman'**s main thesis was quantum systems could not be efficiently imitated on classical systems.**

Why QS?

Simulating of quantum systems

^l **Quantum computers - Universal quantum simulators**

1982 Richard P. Feynmann R.P. Feynman, "Simulating Physics with Computers" , *Int. J. Theor. Phys.* **21**, 467-488, 1982

Can we do it with a new kind of computer – a quantum computer? Now it turns out, as far as I can tell, that you can simulate this with a quantum system, with quantum computer elements. […] I therefore believe it'*s true that with a suitable class of quantum machines you can imitate any quantum system, including the physical world.*

 Quantum simulator ≠ Universal quantum computer

Applications of QS

Physical implementations of QS

核自旋体系

- 核自旋具有较长的消相干时间
- 相当成熟的磁共振技术
- 很好的测试平台

Spin-based QIP

Spintronics

NMR QIP

Different classes of quantum simulations

- **explore new physics (perhaps even trackable classically)**
- **outperform classical computation (address the classically nontrackabkle)**

Quantum harmonic and anharmonic oscillators

Many-fermion system

Quantum spin model (quantum phase transition)

Localization effects by

Quantum walk

Quantum chemistry

Quantum chaos

Paring Hamiltonian

Quantum Tunneling

decoherence *Entropy* 2010, *12*, 2268-2307

Basic principle of QS

QS: a controllable quantum system used to simulate or emulate other quantum systems

Two types:

Digital quantum simulation: to

use qubits to encode the state of the quantum system, "translate" its unitary evolution in terms of elementary quantum gates, and implement them in a circuitbased quantum computer.

Analog quantum simulation: to map the evolution of the system to be

simulated onto the controlled evolution of the quantum simulator

 $H_{\text{sys}} \leftrightarrow H_{\text{sim}}$.

I. M. Georgescu et al., Rev. Mod. Phys., Vol. 86, No. 1, January–March 2014

Basic principle of QS

Main steps

^l *Mapping*

^l *Initialization*

- Direct state construction
- Adiabatic quantum state preparation

^l *Hamiltonian engineering*

- Lloyd's method (Average Hamiltonian theory)
- \bullet Quantum network

^l *Measurement*

- Quantum state tomography (full characterization)
- Phase estimation algorithm (Energy spectrum and eigenstates)
- Specialized measurement scheme to extract the desired observables (e.g., correlation functions)

Mapping

Quantum spin model (Quantum magnets)

$$
H = \sum_{i=1}^{n} B_{i} \sigma_{iz} + \sum_{i < j=1}^{n} \left(J_{ij}^{x} \sigma_{ix} \sigma_{jx} + J_{ij}^{y} \sigma_{iy} \sigma_{jy} + J_{ij}^{z} \sigma_{iz} \sigma_{jz} \right)
$$

External fields Heisenberg couplings

Heisenberg isotropic, Ising, XX, XY, XYZ model

Mapping: A more realistic model in that it treats the spins quantummechanically, by replacing the spin by a quantum operator (Pauli spin-1/2 matrices at spin 1/2).

Hamiltonian Engineering

^l **Quantum Control Model**

$$
U = e^{-iH_I T} \stackrel{\overline{H}_p = \phi H_I \phi^{-1}}{\longrightarrow} V_T = e^{-i\overline{H}_p T} = \prod_k e^{-iH_p t_k} V_k
$$

• Lloyd'**s** method $\hat{H} = \sum \hat{H}_j$ Trotter-Suzuki formula \hat{t}

$$
U(t) = e^{-iHt} = (e^{-iH_1(t/m)}e^{-iH_2(t/m)}\cdots e^{-iH_k(t/m)})^m
$$

+
$$
\sum_{i < j} [H_i, H_j] \frac{t^2}{2m} + \ldots
$$
 (

the number of operations $Op_{\text{Lloyd}} \propto t^2 n g^2/\epsilon$ Polynomial scaling

S. Lloyd. Universal Quantum Simulators. Science, 273(5278):1073–1078, 1996.

Hamiltonian simulation

Example: Simulating many-body interactions

Measurement

Obtaining some properties of the state

Quantum information and many-body physics

 \triangleright Quantum many-body systems on a lattice

$$
\text{Hilbert space: } \mathcal{H} = \bigotimes_{x \in L} \mathcal{H}_x
$$
\n
$$
\text{Hamiltonian: } \hat{H} = \sum_{x \in L} \hat{h}_x
$$

How much entanglement is contained in the ground state? Scaling of ground state entanglement and Quantum phase transition

Quantum degree of freedom per lattice site

A. Osterloh et al., Nature 416, 609 (2002)

Entanglement and QPTs

Change in the ground-state wavefunction in the critical region: the concurrence as a function of **λ.**

XH Peng et al., PRA 72, 052109 (2005)

simulating a quantum magnet

□ 使用压缩量子模拟方法, 将n自旋链的模拟过程压缩 ⾄**log(n)**⽐特空间,⼤⼤减少计算资源

 \triangleright Quantum matchgate circuits

$$
G(A, B) = \begin{pmatrix} p & 0 & 0 & q \\ 0 & w & x & 0 \\ 0 & y & z & 0 \\ r & 0 & 0 & s \end{pmatrix}, \quad A = \begin{pmatrix} p & q \\ r & s \end{pmatrix}, \quad B = \begin{pmatrix} w & x \\ y & z \end{pmatrix}
$$

Theorem 2.1. Consider any matchgate circuit of size N and width n, such that:

(i) the matchgates $G(A, B)$ act on n.n. qubit lines only;

(ii) the input state is any computational basis state $|x1 \cdots xn\rangle$;

(iii) the output is a final measurement in the computational basis on any single qubit line.

Then the output may be classically efficiently simulated

R. Jozsa, et al., Proc. R. Soc. A 466, 809 (2009) B. Kraus, PRL 107, 250503 (2011)

 \triangleright Quantum Ising spin chain

$$
H(J) = \sum_i \sigma_z^i + J \sum_i \sigma_x^i \sigma_x^{i+1}
$$

Ø Jordan–Wigner transformation

Fermionic operators
\n
$$
a_k = Z...Z\sigma_k^{-1}
$$
\n
$$
a_k^{\dagger} = Z...Z\sigma_k^{+1}
$$
\n
$$
\{a_i, a_j\} = 0 \text{ and } \{a_i, a_j^{\dagger}\} = \delta_{ij}1
$$
\n
$$
c_{2k-1} = a_k + a_k^{\dagger}, \quad c_{2k} = (a_k - a_k^{\dagger})/i, \quad k = 1, ..., n
$$
\n
$$
\{c_j, c_l\} \equiv c_j c_l + c_l c_j = 2\delta_{j,l}I, \quad j, l = 1, ..., 2n
$$
\nQuadratic Hamiltonian
\n
$$
H = i \sum_{\mu \neq \nu = 1}^{2n} h_{\mu\nu} c_{\mu} c_{\nu},
$$

hmn is a 2n x 2n matrix of coefficients

Theorem: Let H be any quadratic Hamiltonian *H* and $U = e^{iH}$

$$
U^{\dagger} c_{\mu} U = \sum_{\nu=1}^{2n} R_{\mu\nu} c_{\nu},
$$

R is a real orthogonal matrix of 2n × $R \in SO(2n)$ 2n associated with U.

$$
\begin{array}{ccc}\nU_N...U_1 & \xrightarrow{\hspace{0.5cm}} R_N...R_1 \\
\textbf{2}^n \times \textbf{2}^n & \textbf{2}^n \times \textbf{2}^n\n\end{array}
$$

Symmetry of the Ising model **n is a set of the Ising model** \longrightarrow **n x n Log(n) qubits** Key idea:

block-diagonal matrices R_0, R_l, S , and ρ_{in} **Unitary V** $|0\rangle\langle 0| \otimes O_1 + |1\rangle\langle 1| \otimes O_2$

 $L(J)$ $U(J) = \sqrt{V_0}$ $\prod U_l\sqrt{V_0}^{\dagger}$, **Traditional quantum simulation (n qubits)**

 $R(J) = \sqrt{R_0} \prod_{l=1}^{L(J)} T_l \sqrt{R_0}^{-1}$ **Compressed quantum simulation (log(n) qubits)**

Ground-state magnetization

Z. Li et al., Phys. Rev. Lett. 112, 220501 (2014)

口 使用不同样品, 研究不同尺度自旋链的基态性质

□ Comparison of traditional **simulation to compressed simulation**

 ϵ

 Ω

 H

160.7

 $\mathbf 0$

F

 -194.4

47.6

 Ω

 T_{2} (s)

 1.1

 1.2

 1.3

 ϵ

 H

F

《Phys. Rev. Lett.》 Highlight Article.

Editors' Suggestion PRL

2014年6月6号

Experimental Realization of a Compressed Quantum Simulation of a 32-**Spin Ising Chain**

Zhaokai Li, Hui Zhou, Chenyong Ju, Hongwei Chen, Wengiang Zheng, Dawei Lu, Xing Rong, Changkui Duan, Xinhua Peng, and Jiangfeng Du

Phys. Rev. Lett. **112**, 220501 - Published 2 June 2014

Using a quatum simulator with only five qubits, the simulation of a 32-spin Ising chain is experimentally demonstrated.

Exotic quantum many-body physics

Spin Chain (complex interaction)

Exotic quantum many-body physics **New Physics: New critical phenomena**

One-dimensional chain constructed out of equilateral triangles

A three-spin cluster Hamiltonian that exhibits a novel kind of critical behavior that is not revealed by the traditional approach -- two-point correlation functions.

Pachos and Plenio, PRL 93, 056402 (2004)

Experimental demonstration

New critical phenomena induced by three-body interaction cannot be detected by the standard two-spin correlation.

Entanglement witness operators

Two-spin Ising model

Three-spin Ising model

XH Peng et al., Phys. Rev. Lett. 103, 140501(2009)

Exotic quantum many-body physics

New Physics: Topological orders (new quantum orders)

Kitaev's toric code model

Wen-plaquette model

$$
H = -J\sum_{i} \hat{F}_{i},
$$

$$
\hat{F}_{i} = \sigma_{i}^{x} \sigma_{i+\hat{e}_{x}}^{y} \sigma_{i+\hat{e}_{x}+\hat{e}_{y}}^{x} \sigma_{i+\hat{e}_{y}}^{y}
$$

Topologic orders = pattern of quantum entanglements A. Kitaev, Ann. Phys. 303, 2 (2003); X. G. Wen, PRL*.* 90, 016803 (2003)

Z2 topological orders

Z2 topological orders

Topological orders (TO)

X. G. Wen, Quantum Field Theory of Many-Body Systems

Topological orders (TO)

Problems and challenges

- \triangleright Fractional quantum Hall (FQH) systems is the physical ones naturally existing topological orders.
- \triangleright Instead of naturally occurring physical systems, twodimensional spin-lattice models, including the toric-code model, the Wen-plaquette model, and the Kitaev model on a hexagonal lattice, were found to exhibit Z2 topological orders.
- \triangleright The study of such systems therefore provides an opportunity to understand more features of topological orders and the associated topological QPTs.

Problems and challenges

 \triangleright Previous experiments for the toric-code model: Photon systems [Nature 482, 489-494 (2012), PRL 102, 030502 (2009), New. J. Phys 11, 083010 (2009)]

 NMR systems [PRA 88, 022305 (2013); arXiv:0712.2694v1 (2007)]

 not really realized quantum spin models, rather than directly prepared some specific entangled states with TO

 \triangleright Two major challenges:

- to engineer and control experimentally complex quantum systems with four-body interactions
- to detect efficiently the resulting topologically ordered phases which is different from symmetry-breaking phase transition.

Wen-plaquette model

An exact soluble quantum spin model with Z2 topological orders

$$
H = -J\sum_{i}\hat{F}_{i}, \quad \hat{F}_{i} = \sigma_{i}^{x}\sigma_{i+\hat{e}_{x}}^{y}\sigma_{i+\hat{e}_{x}+\hat{e}_{y}}^{x}\sigma_{i+\hat{e}_{y}}^{y}
$$

 \triangleright For J >0, the ground state is

Beyond Landau's symmetry breaking theory

Question: How to observe in the experiment?

X. G. Wen, PRL*.* **90, 016803 (2003)**

Phase diagram

Transverse Wen-plaquette model

The transition region depends on the value of $\mathsf I$

• The transition becomes narrower and *sharper as g decreases.* • When $g \rightarrow 0$, $Z2B \rightarrow Z2A$ topological order $\omega J = 0$.

Analogy quantum simulation

Validity of TQPT in the small finite-size system

 TOs exist in the Wen-plaquette model with periodic lattice of nite size X. G. Wen, PRL*.* 90, 016803 (2003)

Spin-spin correlations

Validity of TQPT in the small finite-size system

g = 0 Wen-plaquette model

a first-order phase transition which can occur in a finite-size system [Rep. Prog. Phys. 66, 2069 (2003)]

Transverse Wen-Plaquette model: second-order phase transition at J/ $g = 1$ in the thermodynamic limit. In finite-size systems, the sharp quantum phase transition is smoothed into a graduate change in the properties of the ground state, but its effect can still be visible.

Wen-plaquette model

2x2 lattice: the simplest finite system to present such a TQPT

 $H_{Wen}^{4} = -2J(\hat{\sigma}_{1}^{x}\hat{\sigma}_{2}^{y}\hat{\sigma}_{3}^{x}\hat{\sigma}_{4}^{y} + \hat{\sigma}_{1}^{y}\hat{\sigma}_{2}^{x}\hat{\sigma}_{3}^{y}\hat{\sigma}_{4}^{x})$

Ground state has four-fold degeneracy

Transverse Wen-plaquettte model :

$$
H_{tol} = H_{Wen}^4 - g \sum_i^{Ns=4} \hat{\sigma}_i^x
$$

Ground state:

$$
|\psi_g\rangle\approx \left\{ \begin{array}{ll} |\psi_{Z2B}\rangle=|\Phi^+\rangle_{13}|\Phi^+\rangle_{24},\;\; J\ll-g<0 \\ |\psi_{SP}\rangle=|++++\rangle, & J=0 \\ |\psi_{Z2A}\rangle=|\Psi^+\rangle_{13}|\Psi^+\rangle_{24},\;\; J\gg g>0 \end{array} \right.
$$

Experiment

Hamiltonian simulation

Measurement

What is the order parameter?

- **▶ Standard QPT detectors:**
	- derivative of the ground state energy,
	- Block entanglement
	- ground state fidelity
- \triangleright Non-local order parameters
	- Topological entanglement entropy
	- Wilson loop $\hat{W}(C) = \prod_C \hat{\sigma}_i^{\alpha_i}$

For the ground state, the closed-strings are condensed

A. Hamma et al., Phys. Rev. B 77, 155111 (2008)

Experimental results

Entanglement, complementarity and TQPT

A conjecture:

conjecture:
\n
$$
\sum_{m=2}^{n} \tau_m^{(k)} + S_k^2 = 1
$$
\n
$$
\sum_{m=2}^{n} \tau_{m}^{(k)} + S_k^2 = 1
$$
\n
$$
\sum_{n=2}^{n} \tau_{m}^{(k)} + S_k^2 = 1
$$
\n
$$
C_{13}^2(\rho^{exp}) \left| C_{13}^2(\rho^{exp}) \right| C_{14}^2(\rho^{exp}) \left| \tau_2^1 = \sum_{j \neq 1} C_{1j}^2(\rho^{exp}) \right|
$$

Larger QS for TO

- \triangleright From the theoretical view: explore more interesting physical phenomena, such as lattice-dependent topological degeneracy (even x even;even x odd; odd x odd), robust ground state degeneracy, quasiparticle fractional statistics, protected edge states, topological entanglement entropy and so on.
- \triangleright From the experimental view: our experimental methods are universal for larger number of qubits; for the larger systems, quantum simulators will perform more powerfully than classical computers in the research of topological orders and their physics, which cannot be efficiently simulated on classical computers.

Remarks

- Ø **The first experimental implementation of adiabatic transitions between different topological orders by simulating the quantum spin model. This provides an experimental tool for further studies of complex quantum systems.**
- Ø **Our experiments demonstrate the feasibility of small quantum simulators for strongly correlated quantum systems, and the usefulness of the adiabatic method for constructing and initializing a topological quantum memory.**

Collaborate with Prof. S. P. Kou X. H. Peng et al., Phys. Rev. Lett. 113, 080404 (2014)

summary: novel physics

Decoherence ≠ error

 $→$ **how to mitigate it how to exploit it (quantum control)**

 $→$ **how to investigate (mesoscopic) decoherence**

scaling quantum simulations

è**proof of principle on spins**

bridging the gap (proof of principle studies and "**useful**" **QS)**

- $→$ **outperforming classical computation**
- è **deeper understanding of quantum dynamics**
- è **new physical phenomena**

investigate the impact on:

- **Solid state physics (magnets, ferroelectrics, quantum Hall, high T_c) (quantum phase transitions, spin frustration, spin glasses,…)**
- **quantum information processing / quantum metrology**

 - …

Thanks to ……

Collaborators:

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Spin Magnetic Resonance Lab 自旋共振实验室

Thanks for your attention!

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